Pearson Edexcel

Mark Scheme (Results)

Summer 2024

Pearson Edexcel GCE

AS Mathematics (8MA0)

Paper 01 Core Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- □ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	$\frac{2\sqrt{x}}{x^2} \to ...x^{-\frac{3}{2}}$ or $\frac{-3}{x^2} \to ...x^{-2}$	M1	1.1b
	$\int \frac{2\sqrt{x}-3}{x^2}\,dx = \int 2x^{-\frac{3}{2}} - 3x^{-2}\,dx$	A1	1.1b
	$...x^{-\frac{3}{2}} \to ...x^{-\frac{1}{2}}$ or $...x^{-2} \to ...x^{-1}$	dM1	1.1b
	$\int 2x^{-\frac{3}{2}} - 3x^{-2}\,dx = -4x^{-\frac{1}{2}} + 3x^{-1} + c$	A1	1.1b
		(4)	
		(4 marks)	

Notes

M1: For separating the fraction into two separate terms. Award for one correct index (which does not need to be processed. e.g. $...x^{\frac{1}{2}-2}$)

Note $-\frac{3}{x^2}$ is insufficient. They must write as $...x^{-2}$ or may be implied by further work.

Beware of candidates who integrate the numerator and denominator which results in a correct index but is an incorrect method and scores M0A0dM0A0

A1: $2x^{-\frac{3}{2}} - 3x^{-2}$ o.e. where the indices have been processed (may be implied by further work)

dM1: For raising the power by one on at least one term with a correct index. It is dependent on the previous method mark. The index does not need to be processed. e.g. $...x^{-\frac{3}{2}} \to ...x^{-\frac{3}{2}+1}$. It is not for $+c$.

A1: All correct, simplified and on one line including + c. Allow other simplified equivalent terms such eg. $-\frac{4}{\sqrt{x}}$ for $-4x^{-\frac{1}{2}}$ or e.g. $\frac{3}{x}$ for $3x^{-1}$ but do not allow e.g. $+-4x^{-\frac{1}{2}}$ or $-\frac{4}{1}x^{-\frac{1}{2}}$

Award once a correct expression is seen and isw but if there is any additional/incorrect notation and no correct expression has been seen on its own, withhold the final mark.

e.g. $\int -4x^{-\frac{1}{2}} + 3x^{-1} + c\,dx$ or $-4x^{-\frac{1}{2}} + 3x^{-1} + c = 0$ with no correct expression seen earlier are both A0. Ignore $y = ...$

Alternative method: integration by parts

e.g. $\int \frac{2\sqrt{x}-3}{x^2}\,dx = \int (2\sqrt{x}-3)x^{-2}\,dx = -(2x^{\frac{1}{2}}-3)x^{-1} + \int x^{-\frac{3}{2}}\,dx$

$-(2x^{\frac{1}{2}}-3)x^{-1} + \int x^{-\frac{3}{2}}\,dx = -2x^{-\frac{1}{2}} + 3x^{-1} - 2x^{-\frac{1}{2}} + c = -4x^{-\frac{1}{2}} + 3x^{-1} + c$

M1: For attempting integration by parts and achieving the correct structure for the intermediate stage

e.g. $\pm(...x^{\frac{1}{2}} \pm ...)x^{-1} \pm \int x^{-\frac{3}{2}}\,dx$ or equivalent. Indices do not need to be processed

e.g. $x^{-2}\left(...x^{\frac{3}{2}} \pm ...x\right) \pm \int ...x^{-3}\left(...x^{\frac{3}{2}} \pm ...x\right)dx$

A1: $-(2x^{\frac{1}{2}} - 3)x^{-1} + \int x^{-\frac{3}{2}}\,dx$ or $x^{-2}\left(\frac{4}{3}x^{\frac{3}{2}} - 3x\right) - \int -2x^{-3}\left(\frac{4}{3}x^{\frac{3}{2}} - 3x\right)dx$ o.e.

dM1: For completing the integration by parts method achieving at least one term with a correct index (the index does not need to be processed) and terms with the same index do not need to be collected for this mark

A1: As above in the main scheme (see notes)

Question	Scheme	Marks	AOs
2(a)	$\left(f(4)=\right)\ 2\times4^3-3a\times4^2+4b+8a=0$	M1	1.1b
	$128+4b=40a\Rightarrow32+b=10a$ *	A1*	1.1b
		(2)	
(b)	$f(2)=2\times2^3-3a\times2^2+2b+8a=0\Rightarrow8+b=2a$	M1	1.1b
	Solve simultaneously $\Rightarrow a=...$ or $\Rightarrow b=...$	dM1	2.1
	$a=3$ or $b=-2$ or $k=3$	A1	1.1b
	$\left(f(x)=\right)\ (2x+3)(x-4)(x-2)$	A1	1.1b
		(4)	
(c)(i)	3	B1	1.1b
(ii)	12	B1ft	2.2a
		(2)	
			(8 marks)

Notes

(a)

M1: Attempts $f(4)=0$ leading to an equation in *a* and *b* only. Condone slips. The = 0 may be implied by further work for this mark. Attempts using algebraic division score M0A0.

A1*: Simplifies and rearranges to the given answer with no errors. There must be at least one intermediate stage of working between their first expression or equation and the given answer and = 0 must be correctly seen at some point in their solution or at the start e.g. stating $f(4)=0$. Isw if they achieve the given answer but then attempt to make *a* or *b* the subject.
Minimum acceptable is e.g. $128-48a+4b+8a\Rightarrow128-40a+4b=0\Rightarrow32+b=10a$
Note: $128-48a+4b+8a\Rightarrow128+4b=40a\Rightarrow32+b=10a$ is M1A0* (we do not see = 0 correctly at some point or e.g. $f(4)=0$)

(b) **Note that there are many different equations which can be formed.**
Sight of $a=3$ or $b=-2$ or $k=3$ scores the first 3 marks BUT answers with no working – send to review

M1: Attempts $f(2)=0$ to form another equation in *a* and *b*. Does not need to be simplified. Condone slips substituting in 2 (but not if there is a clear intention to substitute in -2) and = 0 may be implied by further work. Accept attempts to divide algebraically by $x-2$: e.g. condone slips but they must have a quadratic quotient $2x^2\pm(...a\pm...)x$ and proceed as far as a remainder of the form $...a\pm...b\pm...$ which is then set equal to 0.

e.g.
$$
\begin{array}{r|llll}
 & 2x^2 & +(-3a+4)x & +(b-6a+8) & \\
\hline
x-2 & 2x^3 & -3ax^2 & +bx & +8a \\
 & 2x^3 & -4x^2 & & \\
 & & (-3a+4)x^2 & +bx & \\
 & & (-3a+4)x^2+(6a-8)x & & \\
 & & & (b-6a+8)x & +8a \\
 & & & (b-6a+8)x-2b+12a-16 & \\
 & & & & -4a+2b+16=0
\end{array}
$$

You may also see an attempt at the grid method:

	$2x^2$	$+(4-3a)x$	$+(8-6a+b)$
x	$2x^3$	$+(4-3a)x^2$	$+(8-6a+b)x$
-2	$-4x^2$	$(-8+6a)x$	$-16+12a-2b$

$-16+12a-2b=8a$

In this method condone slips but they need to proceed as far as a quadratic quotient of the form $2x^2 \pm(...a\pm...)x$ and the bottom right cell of the form $...a\pm...b\pm...$ which is then set equal to $8a$.

Look out for other valid methods such as

- multiplying out $\text{f}(x)=(2x+k)(x-4)(x-2)$ to achieve a cubic where the coefficients of x^2 and x, and the constant term are all in terms of k, and equating coefficients to form two more simultaneous equations
- algebraically dividing $2x^3-3ax^2+bx+8a$ by x^2-6x+8 and setting their remainder equal to zero. Look for a quotient of the form $2x\pm...a\pm...$
- substituting $b=10a-32$ into $\text{f}(x)$ so that the polynomial coefficients are in terms of a only (or equivalently all in b only) and using $\text{f}(2)=0$ to form an equation in one variable.

Send to review if you are unsure.

dM1: Attempts to solve their equations simultaneously to find a value for a or b (or they may proceed directly to finding a value for k). This may be done on a calculator. You do not need to check their method for solving. It is dependent on the previous method mark.

A1: $a=3$ or $b=-2$ or $k=3$

A1: $(\text{f}(x)=)\ (2x+3)(x-4)(x-2)$ (All on one line) (Stating the values of a, b and k alone does not score this mark). Allow to be scored if seen in (c).

Alternative Further Maths method using the sum and product of the roots

M1: $\alpha+\beta+\gamma=\frac{3a}{2}\Rightarrow 4+2-\frac{k}{2}=\frac{3a}{2}$ or $\alpha\beta\gamma=-\frac{8a}{2}\Rightarrow 4\times2\times\left(-\frac{k}{2}\right)=-\frac{8a}{2}$

A1: $\alpha+\beta+\gamma=\frac{3a}{2}\Rightarrow 4+2-\frac{k}{2}=\frac{3a}{2}$ and $\alpha\beta\gamma=-\frac{8a}{2}\Rightarrow 4\times2\times\left(-\frac{k}{2}\right)=-\frac{8a}{2}$

(Less likely but possible to see $\alpha\beta+\beta\gamma+\alpha\gamma=\frac{b}{2}\Rightarrow 4\times2+2\times\left(-\frac{k}{2}\right)+4\times\left(-\frac{k}{2}\right)=\frac{b}{2}$)

dM1A1A1: See notes above

(c)

(i)

B1: 3 (listing the actual roots only is B0)

(ii)

B1ft: 12 only Follow through on their $2x+k\Rightarrow-\frac{3k}{2}$ if $k<-8$

Question	Scheme	Marks	AOs
3(a)	$\overrightarrow{PQ} = (3-9)\mathbf{i} + (-5+8)\mathbf{j}$	M1	1.1a
	$= -6\mathbf{i} + 3\mathbf{j}$	A1	1.1b
		(2)	
(b)	Gradient of $PQ = \dfrac{-5--8}{3-9}\left(=-\dfrac{1}{2}\right)$ and Gradient of $QR = \dfrac{18}{9}(=2)$ or $\lvert\overrightarrow{PQ}\rvert = \sqrt{"(-6)"^2 + "3"^2}\ \left(=3\sqrt{5}\right)$ and $\lvert\overrightarrow{QR}\rvert = \sqrt{9^2+18^2}\ (=9\sqrt{5})$ and $\lvert\overrightarrow{PR}\rvert = \sqrt{"3"^2 + "21"^2}\ (=15\sqrt{2})$	M1	3.1a
	e.g. shows that $-\dfrac{1}{2}\times 2 = -1$ and deduces angle $PQR = 90°$ * or e.g. shows $\lvert\overrightarrow{PQ}\rvert^2 + \lvert\overrightarrow{QR}\rvert^2 = \lvert\overrightarrow{PR}\rvert^2$ and deduces angle $PQR = 90°$ *	A1*	2.4
		(2)	
(c)	Attempts to find the length PQ and at least one of QR or PS using Pythagoras' Theorem correctly e.g. $\lvert\overrightarrow{PQ}\rvert = \sqrt{"(-6)"^2 + "3"^2}$ **and** **either** $\lvert\overrightarrow{QR}\rvert = \sqrt{9^2+18^2}$ **or** $\lvert\overrightarrow{PS}\rvert = \sqrt{"27"^2 + "54"^2}$	M1	2.1
	$\lvert\overrightarrow{PQ}\rvert = "\sqrt{45}"\ \left(=3\sqrt{5}\right)$ **and** **either** $\lvert\overrightarrow{QR}\rvert = \sqrt{405}\ \left(=9\sqrt{5}\right)$ **or** $\lvert\overrightarrow{PS}\rvert = 27\sqrt{5}$	A1ft	1.1b
	e.g. Area $= \dfrac{1}{2}\times\left("9\sqrt{5}" + "27\sqrt{5}"\right)\times "\sqrt{45}"$ or $\dfrac{1}{2}\times 4\times "9\sqrt{5}"\times "3\sqrt{5}"$	dM1	3.1a
	$= 270$	A1	1.1b
		(4)	
		(8 marks)	

Notes

Note that work seen must be used in the relevant part. If there is a lack of labelling of parts then award the marks to the parts which leads to the highest total overall.

(a)

M1: Attempts subtraction either way round (does not need to be evaluated). It cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component or sight of $\mp 6\mathbf{i} \pm 3\mathbf{j}$.

A1: Correct answer. Allow $-6\mathbf{i} + 3\mathbf{j}$ or $\begin{pmatrix}-6\\3\end{pmatrix}$ but do not allow $\begin{pmatrix}-6\mathbf{i}\\3\mathbf{j}\end{pmatrix}$ isw once a correct answer is seen.

(b) Condone lack of labelling / poor notation for lengths/angles provided the intention is clear

M1: Attempts to find the gradient of the line PQ and the gradient of the line QR. If they find the reciprocals of **both** they must be labelled e.g. $\dfrac{dx}{dy}$ o.e. (but not gradient or m)

Do not allow sign slips for this mark. Alternatively they may find the lengths PQ, QR and PR or PQ^2, QR^2 and PR^2

Be aware of Further Maths methods such as attempting the dot product $\begin{pmatrix}-6\\3\end{pmatrix}\cdot\begin{pmatrix}9\\18\end{pmatrix}=(-6\times9)+(3\times18)$

A1*: **Correct working** and conclusion that angle $PQR = 90°$

- Using gradients or their reciprocals they need to state or show that the product is equal to -1 o.e. or refer to the values being negative reciprocals of each other
- Using Pythagoras' Theorem they must state or show that $|\overrightarrow{PQ}|^2+|\overrightarrow{QR}|^2=|\overrightarrow{PR}|^2$
- Using the cosine rule and finding angle $PQR = 90°$
- Using the scalar dot product they must show that $\begin{pmatrix}-6\\3\end{pmatrix}\cdot\begin{pmatrix}9\\18\end{pmatrix}=0$

In all cases there must be some sort of minimal conclusion that angle $PQR = 90°$ e.g. "hence right angle" or if they start with a preamble it is acceptable to state "hence proven", "QED" or a tick. Use of e.g. cosine rule resulting in $90°$ is sufficient.

(c) Condone lack of labelling / poor notation for lengths provided the intention is clear

M1: Correct use of Pythagoras' Theorem to find the length of PQ and at least one of QR or PS. Must be used or seen in (c) to score this mark. Condone working using rounded or truncated values.

A1ft: Correct length of PQ and at least one of QR or PS. Follow through on their vectors for PQ and PS but QR must be $\sqrt{405}$ or equivalent. Lengths do not need to be simplified but they must be exact. Must be used or seen in (c) to score this mark.

dM1: Correct method to find the area of the trapezium. It is dependent on the first method mark and the method to find any lengths must be correct.

This may be achieved by calculating $\frac{1}{2}\times4|QR|\times|PQ|$

Alternatively, they may find the area of a rectangle + triangle so look for:

e.g. $|PQ|\times|QR|+\frac{1}{2}\times(|PS|-|QR|)\times|PQ|=\text{"}\sqrt{45}\text{"}\times\text{"}9\sqrt{5}\text{"}+\frac{1}{2}\times\text{"}18\sqrt{5}\text{"}\times\text{"}\sqrt{45}\text{"}$

Note that there are other combinations of lengths to find the area of a rectangle and either add or subtract triangles as appropriate. Condone working using rounded values.

A1: 270

Alt (c) "Shoelace method" or other methods using position vectors

M1: Correct method to find either the position vector of R or the position vector of S. May be seen as coordinates. Check any diagram drawn.

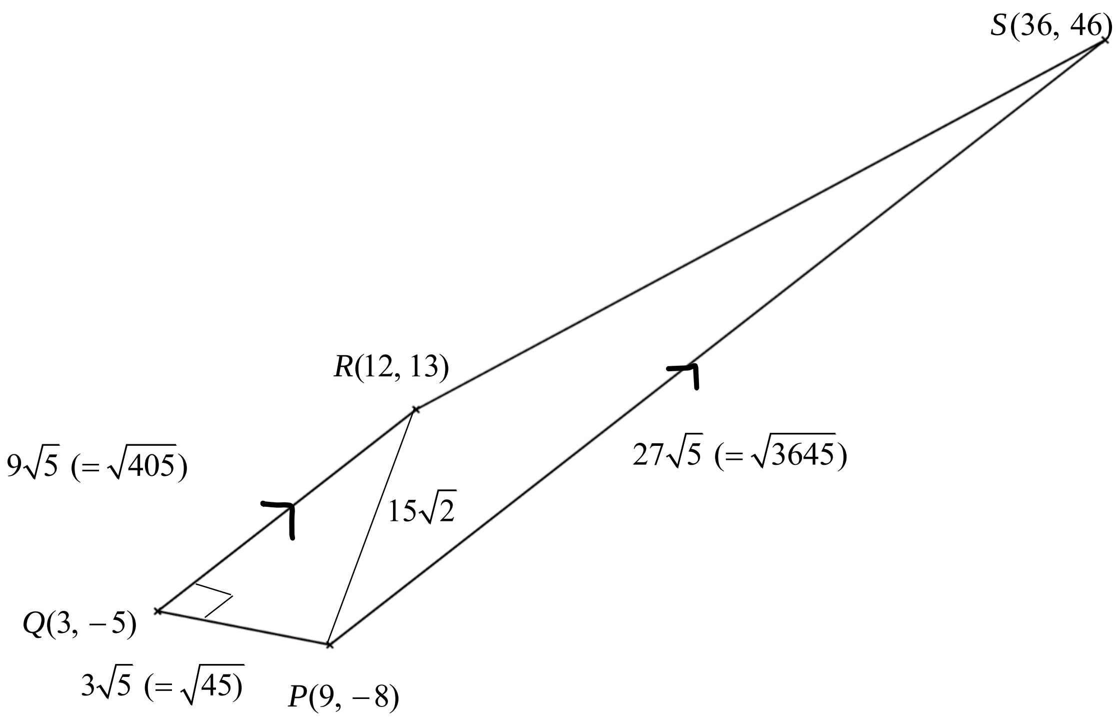
A1: R has position vector $12\mathbf{i}+13\mathbf{j}$ and S has position vector $36\mathbf{i}+46\mathbf{j}$ (or equivalent). May be seen as coordinates. Check any diagram drawn.

dM1: Correct method to find the area of the trapezium via the "shoelace" method:

$$\frac{1}{2}\begin{vmatrix}9 & -8\\3 & -5\\12 & 13\\36 & 46\\9 & -8\end{vmatrix}=\frac{1}{2}|(9\times(-5)+3\times13+12\times46+36\times(-8))-(3\times(-8)+12\times(-5)+36\times13+9\times46)|$$

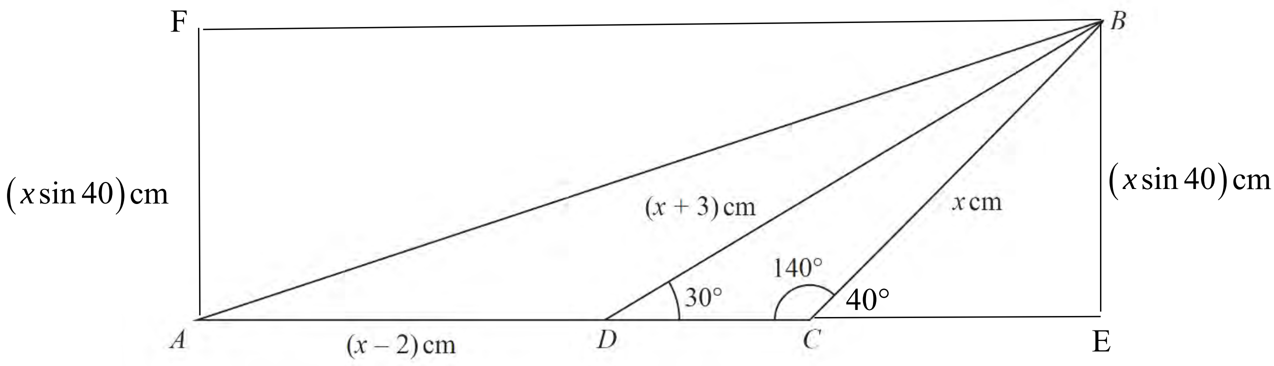
$$=\frac{1}{2}|258-798|$$

A1: 270


S(36, 46)
R(12, 13)
$9\sqrt{5}$ $(=\sqrt{405})$
$27\sqrt{5}$ $(=\sqrt{3645})$
$15\sqrt{2}$
Q(3, −5)
$3\sqrt{5}$ $(=\sqrt{45})$
P(9, −8)

Question	Scheme	Marks	AOs
4(a)	$\dfrac{x}{\sin 30°}=\dfrac{x+3}{\sin 140°}$	M1	1.1b
	$2x\sin 140 - x = 3$	dM1	3.1a
	$x = 10.5$ *	A1*	1.1b
		(3)	
(b)	$AB^2 = 8.5^2 + 13.5^2 - 2\times 8.5\times 13.5\times\cos\text{"}150\text{"}$ $\Rightarrow AB = ...$	M1	3.1a
	$AB =$ awrt 21.3 (cm)	A1	1.1b
		(2)	
			(5 marks)

Notes



(a) **Do not allow verification attempts using $x = 10.5$ by substituting into formulae**

M1: Recognises the need to apply the sine rule and attempts to use it with the sides in the correct positions to form an equation in x. Alternatively, uses trigonometric ratios twice to form an equation in x

e.g. $BE = x\sin 40° \Rightarrow \sin 30° = \dfrac{x\sin 40°}{x+3}$ or e.g. $AF = x\sin 40° \Rightarrow \cos 60° = \dfrac{x\sin 40°}{x+3}$

May also see an attempt at Pythagoras' Theorem:

$BD^2 = BE^2 + DE^2 \Rightarrow (x+3)^2 = \left((x+3)\cos 30°\right)^2 + (x\sin 40°)^2$

dM1: Attempts to rearrange the equation by collecting terms in x on one side of the equation and non x terms on the other. It is dependent on the previous method mark. Condone sign slips only in their rearrangement but allow miscopying/arithmetical slips when rounding. Do not allow this mark to be scored for proceeding directly to a numerical value for x.
In the attempt using Pythagoras' Theorem, look for an attempt to multiply out and rearrange to form a three-term quadratic.

A1*: Achieves 10.5 with sufficient working shown. Condone invisible brackets to be recovered. **They must have achieved either a correct expression for x** or $x(2\sin 140° - 1) = 3$ o.e. before stating 10.5 or better. It is acceptable to use rounded decimals for trigonometric values provided they are correct to at least 3sf. e.g. $\sin 140° = 0.64278...$

e.g. $2x = \dfrac{x+3}{\sin 140°} \Rightarrow x = \dfrac{1.5}{\sin 40° - 0.5} = 10.5$ scores M1dM1A1* (allow 0.5 in their initial equation for sin30 and allow interchanging of equivalent angles such as sin140 and sin40) e.g. $x\sin 140 = (x+3)\sin 30° \Rightarrow x = \dfrac{3\sin 30°}{0.643 - \sin 30°} = 10.5$ scores M1dM1A1*

e.g. $\dfrac{x}{\sin 30°} = \dfrac{x+3}{\sin 140°} \Rightarrow x\sin 140° = (x+3)\sin 30° \Rightarrow x = 10.505....$ is M1dM0A0* (insufficient working shown as they do not reach an expression for x or $x(2\sin 140° - 1) = 3$ o.e.)

In the method using Pythagoras' Theorem usual rules apply for solving a quadratic but it cannot be awarded for proceeding to the answer via a calculator (e.g. must see use of the quadratic formula or completing the square).

(b)

M1: Applies the cosine rule correctly using 150° (or 180°-30° seen if incorrect) for angle *ADB* with the correct numerical lengths by attempting to substitute in $x = 10.5$ or better and proceeds to obtain a value for *AB*. Look for other methods such as finding lengths *DC* and *CE* followed by using trigonometry or Pythagoras' Theorem on triangle *ABE*. (see below)

A1: awrt 21.3 (cm) Units not required but if they are given they must be correct. (Using full calculator display 21.29959497/using $x = 10.5$ gives 21.28…)

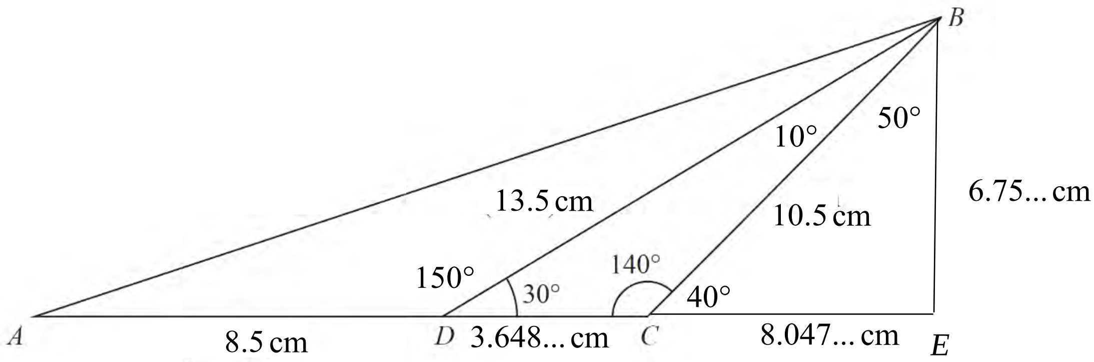
Alt (b)

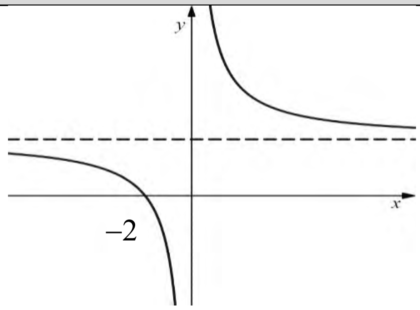
Note that there are various longer methods to finding *AB*. General principles of marking alternative methods:

M1: A complete attempt to find a value for *AB*, but condone slips. The angles and lengths in formulae should be in the correct positions, but allow slips when rearranging or calculating values.

e.g. $BE = 6.75...$, $AE = 8.5 + 3.648... + 8.047... = 20.... \Rightarrow AB = \sqrt{6.75...^2 + 20...^2} = ...$

A1: awrt 21.3 (cm) Units not required but if they are given they must be correct. (Using full calculator display 21.29959497/using $x = 10.5$ gives 21.28…)



Question	Scheme	Marks	AOs
5(a)(i)		M1 A1	1.1b 1.1b
(ii)	Asymptotes: $y=3,\quad x=0$	B1	1.1b
		(3)	
(b)	e.g. $\dfrac{6}{x}+3=3x^2-4x-10 \Rightarrow 6+3x=3x^3-4x^2-10x$	M1	1.1b
	$3x^3-4x^2-13x-6=0$ *	A1*	1.1b
		(2)	
(c)	$x^2-2x\pm\ldots$ e.g. $3x+2\overline{\big)3x^3-4x^2-13x-6}$	M1	3.1a
	$x^2-2x-3\ (=0)$ or $3x^2-6x-9\ (=0)$	A1	1.1b
	e.g. $(3x+2)(x-3)(x+1)=0 \Rightarrow x=\ldots$	dM1	1.1b
	$x=-1,\ x=3$	A1cso	1.1b
		(4)	
			(9 marks)

Notes

(a)(i)

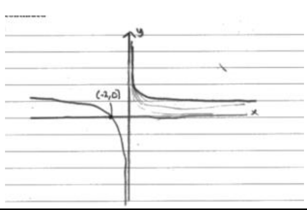
M1: Either

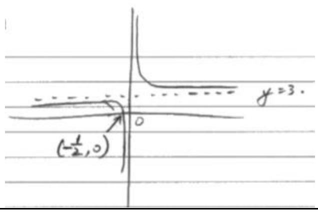
- A correct positive reciprocal graph shape with both branches anywhere on a set of axes (ignore any dashed lines / scale and just look at the graph shape drawn)
- $x=-2$ is the only point of intersection of their graph with the axes. Condone labelled as $(0,-2)$. Allow if they state the coordinates $(-2, 0)$ next to the graph instead but not just $x=-2$

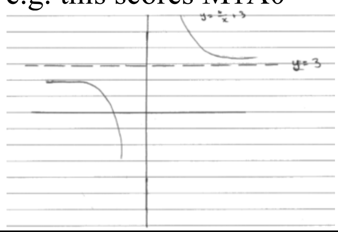
Condone poor curvature provided there is not an intention to draw a maximum or minimum for either of the branches. Condone where the branches approach an undrawn horizontal asymptote if the left-hand branch is slightly above the right-hand branch as long as it does not appear intentional.

A1: Fully correct graph with an intersection labelled at $x=-2$ on the graph (condone labelled as $(0,-2)$). If two horizontal or vertical asymptotes are drawn or a vertical asymptote which is not the *y*-axis then this mark cannot be scored. (unless the vertical asymptote is labelled as 0 or $x=0$) Allow if they state the coordinates $(-2, 0)$ next to the graph instead but not just $x=-2$

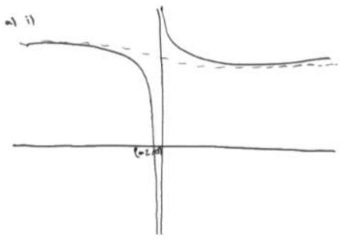
e.g. this scores M1A1 e.g. this scores M1A0 e.g. this scores M1A0

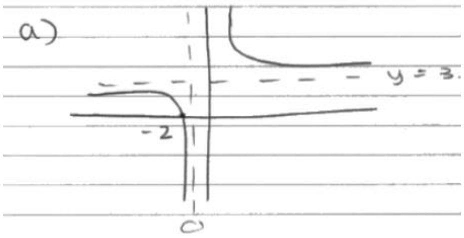






e.g. this scores M1A1 e.g. this scores M1A1 (condone the vertical asymptote drawn as it is labelled as 0 (for $x = 0$))





(ii) **If no graph is drawn then this mark cannot be scored**

B1: $y = 3$, $x = 0$ cao (not $x \neq 0$) which must be correct for their graph. Check by the question but if there is a contradiction then the equations stated in (ii) of the work takes precedence. Cannot be for just stating 0 underneath the *y* axis and 3 on a dashed horizontal line if present.

(b)

M1: Sets $\frac{6}{x} + 3 = 3x^2 - 4x - 10$ and rearranges to form a cubic equation. Terms do not need to be collected or all on one side.

A1*: $3x^3 - 4x^2 - 13x - 6 = 0$ with no errors seen. There must be at least one stage of intermediate working before achieving the given answer.

e.g. $\frac{6}{x} + 3 = 3x^2 - 4x - 10 \Rightarrow 3x^3 - 4x^2 - 13x - 6 = 0$ is M1A0*

e.g. $\frac{6}{x} + 3 = 3x^2 - 4x - 10 \Rightarrow 6 + 3x = 3x^3 - 4x^2 - 10x \Rightarrow 3x^3 - 4x^2 - 13x - 6 = 0$ is M1A1*

(c) **Note that proceeding to three linear factors with no working is M0A0dM0A0. A quadratic factor is required first.**

M1: Attempts to algebraically divide $3x^3 - 4x^2 - 13x - 6$ by $3x + 2$ or compare coefficients to achieve $(x^2 - 2x \pm ....)$ or $(x^2 \pm ...x - 3)$ (or via inspection)

Alternatively, attempts to algebraically divide $3x^3 - 4x^2 - 13x - 6$ by $x + \frac{2}{3}$ or compare coefficients to achieve $(3x^2 - 6x \pm ...)$ or $(3x^2 \pm ...x - 9)$ (or via inspection)

May use the grid method so they must proceed as far as the diagonal for x^2 being correct and achieving $x^2 - 2x$ on the top. (They should reach the same point if dividing by $x + \frac{2}{3}$)

	x^2	$-2x$
$3x$	$3x^3$	$-6x^2$
2	$2x^2$	

Any attempts to use $(x+1)$ or $(x-3)$ rather than $(3x+2)$ from the given root please send to review.

A1: $x^2 - 2x - 3$ $(=0)$ or $3x^2 - 6x - 9$ $(=0)$

dM1: Attempts to solve their resulting quadratic by factorising, using the formula or completing the square. Usual rules apply for solving a quadratic by either of these methods. They cannot simply write down the roots. It is dependent on the previous method mark.

Note $(3x^2 - 6x - 9) \Rightarrow (x-3)(x+1) \Rightarrow x = ...$ is dM0A0

A1cso: $x = -1$, $x = 3$ only (ignore any reference to $x = -\frac{2}{3}$) Condone invisible brackets if recovered or implied by further work. Withhold this mark if they reject one of the two solutions.

Question	Scheme	Marks	AOs
6(a)	e.g. ${}^{12}C_1 \times a = -\frac{15}{2}$	M1	1.1b
	$12a = -\frac{15}{2} \Rightarrow a = -\frac{5}{8}$ *	A1*	1.1b
		(2)	
(b)	${}^{12}C_2 \times a^2 = k \Rightarrow k = ...$	M1	1.1b
	$k = \frac{825}{32}$	A1	1.1b
		(2)	
(c)	$1 - \frac{5}{8}x = \frac{17}{16} \Rightarrow x\ (= -0.1) \Rightarrow 1 - \frac{15}{2} \times ("-0.1") + "\frac{825}{32}" \times ("-0.1")^2$	M1	3.1a
	$=$ awrt 2.0078	A1	1.1b
		(2)	
		(6 marks)	

Notes

(a)

M1: Correct equation to find a e.g. $12a = -\frac{15}{2}$ or ${}^{12}C_1 \times a = -7.5$ or $\frac{12!}{11!}a = -\frac{15}{2}$.

Condone if x is present on both sides of their equation.

Also allow attempts using $a = -\frac{5}{8}$ e.g. ${}^{12}C_1 \times -\frac{5}{8} = -\frac{15}{2}$

A1*: Rearranges to achieve the given answer with no errors and sufficient steps shown i.e. the binomial coefficient must have been evaluated first.

e.g. $\binom{12}{1} \times a = -\frac{15}{2} \Rightarrow 12a = -\frac{15}{2} \Rightarrow a = -\frac{5}{8}$ is M1A1*

Minimum acceptable $12a = -\frac{15}{2} \Rightarrow a = -\frac{5}{8}$ is M1A1*

${}^{12}C_1 \times a = -7.5 \Rightarrow a = -\frac{5}{8}$ is M1A0*

Do not penalise if solutions contain x as well e.g. $12ax = -\frac{15}{2}x \Rightarrow a = -\frac{5}{8}$ is M1A1*

Attempts using $a = -\frac{5}{8}$ must show the binomial coefficient evaluated and there must be a conclusion such as "hence shown", QED

$12 \times -\frac{5}{8}x = -\frac{15}{2}x$ with e.g. hence shown is M1A1*

We do not need to be concerned with any workings related to other terms.

(b) If the expression is seen in (a) it must be used in (b) to score

M1: Correct expression or equation to find k and proceeds to find a value for k e.g. $66a^2 = k \Rightarrow k = ...$. May be implied by a correct answer or $\frac{825}{32}x^2$ .Allow use of $\frac{5}{8}$ for a.

A1: $\frac{825}{32}$ o.e. e.g. 25.78125. isw if they round after a correct answer is seen.

Do not accept $\frac{825}{32}x^2$ but allow the coefficient to be circled or underlined to identify the answer.

(c)

M1: A correct strategy to find the appropriate value of x e.g. $1-\frac{5}{8}x=\frac{17}{16} \Rightarrow x=... (=-0.1)$

and substitutes their value of *x* into $1-\frac{15}{2}x+kx^2$ using their value for *k*. Their $x=-0.1$ embedded in the expression is sufficient. Accept a sign slip on the substitution of their *x* and/or *k* provided the embedded values are seen.

Maybe implied by their value for the expression (you may need to check this on your calculator if the substitution is not seen)

Alternatively, attempts $\left(1+\frac{1}{16}\right)^{12}=1+12\times\frac{1}{16}+\frac{12\times11}{2}\times\left(\frac{1}{16}\right)^2$

Do not withhold this mark if they attempt incorrectly to find additional terms of the binomial expansion.

A1: awrt 2.0078 (full answer on calculation is 2.0078125 or allow $\frac{257}{128}$).

This value with no working seen can score both marks. isw if they round after a correct answer is seen.

Note that if they find the term in x^3 this is $-\frac{6875}{128}x^3$ which using $x=-0.1$ in the expansion up to and including this would give awrt 2.0615 which can score both marks.

Note that 2.06988999…is the value from the calculator for $\left(\frac{17}{16}\right)^{12}$ which is likely to score M0A0 (unless some method is shown for finding the value of *x* and substituting in)

Question	Scheme		Marks	AOs
7(a)	$\log_{10} a = 3.3$ or $\log_{10} b = \frac{2.1-3.3}{6} \left(=-0.2\right)$	e.g. $\log_{10} P = 3.3 - 0.2x$ or $\log_{10} P = 3.3 + \frac{2.1-3.3}{6}x$	M1	1.1b
	$a = 10^{3.3}$ or $b = 10^{-0.2}$		A1	1.1b
	$a = 10^{3.3}$ and $b = 10^{"-0.2"}$	$P = 10^{3.3-0.2x} = 10^{3.3} \times 10^{-0.2x}$	dM1	3.1a
	$P = 1995 \times 0.6310^{x}$		A1	3.3
			(4)	
(b)	The **concentration** (in parts per million) **1 km** from the chimney		B1	3.2a
			(1)	
				(5 marks)

Notes

(a)

M1: Attempts an equation in *a* or *b*. Score for $\log_{10} a = 3.3$ or $\log_{10} b = \frac{2.1-3.3}{6} \left(=-0.2\right)$

Condone an incorrectly evaluated gradient provided $\frac{2.1-3.3}{6}$ o.e. was attempted (may be seen as two simultaneous equations). Alternatively, forms a correct linear equation in $\log_{10} P$ and *x*. Do not penalise if base 10 is missing.

May be implied by a correct unsimplified value for *a* or *b*. (which could be truncated rather than rounded for *b*)

A1: A correct unsimplified value for *a* or *b*. This may be within the linear equation in the alternative method e.g. $P = 10^{3.3} \times 10^{-0.2x}$

dM1: A correct method to find unsimplified values for *a* and *b*. Allow use of their -0.2 found from a correct attempt at the gradient of the line. Alternatively, correctly uses laws of indices to achieve $P = 10^{3.3} \times 10^{"-0.2"x}$. It is dependent on the previous method mark. May be implied by their final correct equation.

A1: Complete equation with *a* = awrt 1995 and *b* = awrt 0.6310 (condone 0.631)

(b)

B1: Must refer to **concentration** (or **e.g. parts per million**) and **1km** o.e.
Condone use of emitted for measured e.g. "concentration of smoke particles emitted 1km from the chimney"
Do not accept "**amount** of smoke particles" or referring to when $x = 1$ (not in context)

Question	Scheme	Marks	AOs
8(a)	$\left(\frac{dy}{dx}=\right) 3x^2-14=-2$ or $\frac{d}{dx}(\pm(\text{curve-line}))=\pm(3x^2-12)=0$	M1	3.1a
	e.g. $3x^2-14=-2 \Rightarrow 3x^2=12 \Rightarrow x=...$	dM1	1.1b
	$x=2$ only *	A1*	2.2a
		(3)	
(b)	e.g. Substitutes $x=-4$ into $y=x^3-14x+23$ and $y=-2x+7$	M1	1.1b
	Correct solution + conclusion * (see notes)	A1*	2.4
		(2)	
(c)	$\int\left(x^3-14x+23\right)\,dx=\frac{x^4}{4}-7x^2+23x\quad(+c)$	M1 A1	1.1b 1.1b
	$=\left(\frac{2^4}{4}-7\times2^2+23\times2\right)-\left(\frac{(-4)^4}{4}-7(-4)^2+23(-4)\right)$	dM1	1.1b
	Area $=\int_{-4}^{2}\left(x^3-14x+23\right)\,dx-\frac{1}{2}\times(15+3)\times(2--4)$ $=\left(\frac{2^4}{4}-7\times2^2+23\times2\right)-\left(\frac{(-4)^4}{4}-7(-4)^2+23(-4)\right)-54$	dM1	3.1a
	Area $=108$ *	A1*	2.1
		(5)	
Alt(c)	$\int\left(x^3-14x+23\right)-(-2x+7)\,dx=\int\left(x^3-12x+16\right)\,dx$ $=\frac{x^4}{4}-6x^2+16x\quad(+c)$	M1 A1	3.1a 1.1b
	Area $=\left(\frac{2^4}{4}-6\times2^2+16\times2\right)-\left(\frac{(-4)^4}{4}-6(-4)^2+16(-4)\right)$	dM1 dM1	1.1b 1.1b
	Area $=108$ *	A1*	2.1
			(10 marks)

Notes

(a) On EPEN this is M1A1A1* we are marking this as M1dM1A1*

M1: Differentiates the cubic to achieve px^2+q and sets equal to -2. Note setting the linear and cubic equations equal to each other and solving is M0 (no use of calculus) but, they may differentiate $\pm(\text{cubic-line})$ to achieve px^2+q and set equal to 0 which is M1. Alternatively, in either approach, differentiates to achieve px^2+q and substitutes in $x=2$

dM1: Proceeds from their equation to find a real value for x with at least one intermediate stage of working. e.g. $px^2=-q \Rightarrow x=...$ or e.g. via factorisation

$(3)(x+2)(x-2)=0 \Rightarrow x=...$ Accept proceeding directly to $x=\sqrt{\frac{14-2}{3}}$ or ±2

In the alternative, it is for substituting in 2 into their $3(2)^2-14=...$ or $3(2)^2-12=...$

A1*: 2 only (the solution of -2 does not need to be found) ±2 without selecting 2/ rejecting -2 is A0*. In the alternative they conclude that $x=2$.

(b) Work may be seen in (a) but must be used in (b) to score

M1: Mark the general method / do not be concerned by slips in their working. Either e.g.

- substitutes $x=-4$ into $y=x^3-14x+23$ and $y=-2x+7$. Sight of -4 embedded in the equations is sufficient.
- substitutes $x=-4$ into one equation to find *y* and then uses this value to find *x* in the other equation.
- equates the cubic and the linear expressions, rearranges (= 0) and then
  - o substitutes $x=-4$ into the equation or expression
  - o factorises using $(x-2)^2$ to find a linear factor (e.g. inspection / division)
  - o algebraically divides by $(x+4)$ at some point to get a remainder of 0
  - o factorises using $(x-2)$ to find a quadratic factor which they attempt to factorise

A1*: **A correct solution with conclusion** but allow recovery of missing/invisible brackets.
Either
- finds $y=15$ when $x=-4$ for both equations
- finds $y=15$ using one equation and uses this to find that $x=-4$ for the other
- verifies that $x=-4$ is a solution of the cubic – linear = 0 with no errors seen.
- solves cubic – linear = 0 and finds the linear factor $(x+4)$ leading to the root $x=-4$

There must be a conclusion: if they find $x=-4$ then a tick, QED, underline, proven. If they find e.g. $y=15$ for both they must state $x=-4$ e.g."(same *y* values so) $x=-4$"
Note that stating the coordinates $(-4, 15)$ is insufficient.

SC: $-2x+7=x^3-14x+23 \Rightarrow x^3-12x+16\ (=0) \Rightarrow x=-4, 2$ so $x=-4$ scores M1A0*

(c) No integration seen (using the integration button on a calculator) will score 0 marks

M1: Integrates the equation of the curve or in the alternative method it is for integrating curve-line and achieving at least two terms with a correct index out of $...x^4 \pm ...x^2 \pm ...x$
Also look out for a transformation approach translating the cubic down 3 units and integrating $\int\left(x^3-14x+20\right)\mathrm{d}x=\frac{x^4}{4}-7x^2+20x$

A1: $\frac{x^4}{4}-7x^2+23x$ or $\frac{x^4}{4}-6x^2+16x$ (ignore any constant of integration). Allow unsimplified but the indices must be processed. Look out for $\frac{x^4}{4}-7x^2+20x$ if the cubic has been translated down 3 units. Do not penalise poor notation for this mark.

dM1: Correct limits of 2 and -4 used for their integral (which may be for the line or curve).
It is dependent on the previous method mark. Evidence of substituting in 2 and -4 into their integrated expression must be seen. (e.g. $22-(-140)$). In the alternative method the dM1 (and next dM1) are scored at the same time for substituting in the correct limits.

dM1: Correct strategy to find the shaded area. **It is dependent on the first method mark only but the limits must be correct**. In the alternative method this mark is scored for sight of 2 and -4 for their integral which do not have to be substituted in.
The method to find any areas e.g. trapezium, rectangle, triangle must be correct and the method of adding or subtracting to find the shaded area must be correct. Condone slips in evaluating. Note using the transformation approach they will need to subtract the area of the triangle $0.5\times6\times(15-3)$

A1*: 108 following from a rigorous argument showing all stages and correct notation used when integrating (**the integral sign and the dx either side of their expression seen at least once** and should not be present after integration has been completed)

Question	Scheme	Marks	AOs
9(a)	$2p$	B1	1.1b
		(1)	
(b)	$\log_a 100 = \log_a 4 + \log_a 25$	M1	1.2
	$\log_a 16^{\frac{1}{2}} + \log_a 25 = \frac{1}{2}p + q$	A1	1.1b
		(2)	
(c)	e.g. $\log_a 80 \times \log_a 3.2 = (\log_a 16 + \log_a 5) \times (\log_a 16 - \log_a 5)$	M1	3.1a
	$\left(p + \frac{1}{2}q\right) \times \left(p - \frac{1}{2}q\right)$ or $p^2 - \frac{1}{4}q^2$	A1	1.1b
		(2)	
			(5 marks)

Notes

(a)

B1: $2p$ o.e.

(b)

M1: Uses the laws of logs to write $\log_a 100$ correctly as a sum of logs

e.g. $\log_a 100 = \log_a 4 + \log_a 25$ e.g. $\log_a 100 = \log_a 4 + \log_a a^q$

e.g. $\log_a 100 = 2\log_a 2 + 2\log_a 5$

e.g. $\log_a 100 = \frac{1}{2}\log_a 16 + \frac{1}{2}\log_a 625$

e.g. $\log_a 100 = \log_a 50 + \log_a 2$

May also be implied by expressions in p or q or a mixture of both

e.g.

$(p + q =) \log_a 16 + \log_a 25 = \log_a 400 = \log_a 4 + \log_a 100 \Rightarrow p + q - \log_a 4 = \log_a 100$

Look out for more complex versions of above e.g. e.g. $\log_a 100 = \log_a 4 + \log_a a^q$ which are acceptable.

Do not penalise the omission of base a

A1: $\frac{1}{2}p + q$ o.e. Correct answer scores full marks but withhold this mark if incorrect log work is seen e.g. $\log_a 100 = \log_a 4 \times \log_a 25$ or $\log_a 100 = 4\log_a 25$

(c)

M1: Uses both the addition and subtraction laws of logs to write the **full expression** of $\log_a 80 \times \log_a 3.2$ correctly in terms of any of the following:

- $\log_a 16 \ (= p)$
- $\log_a 4 \left(= \frac{p}{2}\right)$
- $\log_a 2 \left(= \frac{p}{4}\right)$
- $\log_a 5 \left(= \frac{q}{2}\right)$
- $\log_a 25 \ (= q)$

e.g. $\log_a 80 \times \log_a 3.2 = (2\log_a 4 + \log_a 5) \times (\log_a 16 - \log_a 5)$

You may see other viable solutions using e.g. $\log_a \frac{1}{5}$, $\log_a \frac{1}{4}$ but they will need to proceed to either e.g. $\log_a 5$ (an integer value) or proceed to an expression in terms of p or q

Do not penalise the omission of base a.

May be implied by equivalent expressions in p or q or a mixture of both.

A1: $\left(p + \frac{1}{2}q\right) \times \left(p - \frac{1}{2}q\right)$ or $p^2 - \frac{1}{4}q^2$ o.e. (does not need to be simplified)

Correct answer scores full marks but withhold this mark if incorrect log work is seen in their solution. isw once a correct answer in terms of p and q is seen.

Question	Scheme	Marks	AOs
10(a)	$\text{tangent} = -2 \rightarrow \text{normal} = \frac{1}{2}$	B1	2.2a
	$\frac{k^2-2k-k-8}{3--1}$	M1	1.1b
	$\frac{1}{2} = \frac{k^2-2k-k-8}{3--1} \Rightarrow \frac{1}{2} = \frac{k^2}{4} - \frac{2k}{4} - \frac{k}{4} - 2$	dM1	1.1b
	$\Rightarrow k^2 - 3k - 10 = 0$ *	A1*	2.1
		(4)	
(b)	$k = 5$	B1	2.3
	y coordinate of P is 13	B1ft	1.1b
	Attempts PQ $\left(= \sqrt{(3--1)^2 + ("25-15-8")^2}\right)$ (or PQ^2)	M1	3.1a
	$(x+1)^2 + (y-13)^2 = 20$	A1	1.1b
		(4)	
			(8 marks)

Notes

(a) Attempts to implicitly differentiate the equation of the circle please send to review

B1: Deduces that the gradient between P and Q is $\frac{1}{2}$. Condone this mark to be scored for sight of $\frac{1}{2}$ o.e which may be on the diagram / by the question. May be implied by further work or seen in an equation.

Look out for e.g. $\frac{-(3--1)}{k^2-2k-k-8} = -2$ **o.e. (negative reciprocal of the gradient *PQ* set equal to** -2**) which implies this mark**.

M1: Attempts to find an expression for the gradient of PQ in terms of k (may be implied) Score for the expression which may be seen in their working. Condone one sign slip in their expression. May be seen in $(y_2 - y_1) = m(x_2 - x_1)$ which does not need to be rearranged.

dM1: Sets their gradient of PQ equal to $\frac{1}{2}$ **(or the negative reciprocal of the gradient *PQ* set equal to** -2**)** and proceeds to a quadratic so that the fraction for the gradient has been split up into separate terms.
e.g. $k^2 - 2k - k - 8 = 2$

Terms do not need to be collected for this mark and they do not all need to be on the same side of the equation. Coefficients do not need to be integers.
Condone slips in their rearrangement.
It is dependent on the first method mark.
If using $(y_2 - y_1) = m(x_2 - x_1)$ the mark would be scored substituting $m = \frac{1}{2}$

A1*: Achieves given answer with no errors seen. Must see = 0. They must have achieved a quadratic which is not the given answer before proceeding to the given answer.

Alt(a) forming/using linear equations with the coordinates of *P* and *Q*

B1: As above in the main scheme / notes

M1: Uses one pair of coordinates to form the equation of a straight line with a gradient of $\frac{1}{2}$

e.g. $k^2-2k=\frac{3}{2}+c$ or $k+8=-\frac{1}{2}+c$ (Condone one sign slip)

e.g. $y-k-8=\frac{1}{2}(x+1)$

May use the coordinates for *Q* to form the equation of a straight line with a gradient of -2

dM1: Uses both pairs of coordinates and proceeds via a correct method to an equation in *k* only

e.g. $k+8=-\frac{1}{2}+c\Rightarrow c=k+8.5$ $\quad k^2-2k=\frac{3}{2}+c\Rightarrow k^2-2k=\frac{3}{2}+"k+8.5"$

Condone slips in their rearrangement.
You will need to look carefully at how they use the coordinates and equations.
It is dependent on the first method mark.

A1*: As above in the main scheme / notes

(b)

B1: Deduces that *k* is 5 only. May be seen by the question or in (a) which is fine but if both roots are found then 5 must be selected/used in further work or the negative root is rejected. May be implied by the *y* coordinate of *P*
Note that B0B1ftM1A0 is possible

B1ft: *y* coordinate of *P* is 13 Check on the diagram or may be implied by further work such as in the equation of the circle. If there is a contradiction between the diagram and the main body of the work then the main body of the work takes precedence.

Follow through on their positive value of *k* (add 8 to their value for *k*). Ignore if they find a value for *P* using their negative value of *k* as well

M1: Finds the distance *PQ* (or PQ^2) using their *y* coordinates from their chosen value of *k* (which may be negative). The value of *k* must be consistently used to find *P* and *Q*.

If they find the distance *PQ* twice (using each value of *k* to find coordinates of *P* and *Q*) then condone this mark to be scored.

The expression is sufficient, but they must be attempting $\sqrt{(3--1)^2+(y_2-y_1)^2}$ o.e. (or $(3--1)^2+(y_2-y_1)^2$ o.e). May be implied by sight of 20 or $\sqrt{20}$

A1: $(x+1)^2+(y-13)^2=20$ only or equivalent e.g. $x^2+y^2+2x-26y+150=0$

Question	Scheme	Marks	AOs
11(a)	At $t=0$, $V_A = 100+20=120 \Rightarrow p = 2\times"120"$	M1	1.1b
	$(p=)\ 240$	A1	1.1b
		(2)	
(b)	$...\mathrm{e}^{0.04T} = ...\mathrm{e}^{-0.02T}$	M1	3.1b
	$0.8\mathrm{e}^{0.04T} = "4.8"\mathrm{e}^{-0.02T}$	A1ft	1.1b
	$...\mathrm{e}^{0.04T} = ...\mathrm{e}^{-0.02T} \Rightarrow \mathrm{e}^{\pm 0.06T} = ...$	dM1	3.1a
	$(T=)$ awrt 29.9 (months)	A1cso	1.1b
		(4)	
		(6 marks)	

Notes

(a)

M1: Attempts to find the price per gram of metal *A* at $t=0$, and then doubles this to find the value of *p*. Can be implied by 240. An expression must be evaluated to score this mark

A1: 240 only (withhold this mark if they find $V_B = 240$ but proceed to state *p* as a different value)

(b) Candidates who state that $\frac{\mathrm{d}V_B}{\mathrm{d}t} = "4.8"\mathrm{e}^{-0.02T}$ can still score full marks question (they had already determined that they needed to take the modulus of the gradient function)

May be in terms of *t* or *T*

M1: Attempts to set $\pm\frac{\mathrm{d}V_A}{\mathrm{d}t}$ equal to $\pm\frac{\mathrm{d}V_B}{\mathrm{d}t}$. Look for an equation of the form $p\mathrm{e}^{0.04T} = q\mathrm{e}^{-0.02T}$ where *p* and *q* are constants or may be implied by further work. e.g. $\mathrm{e}^{\pm 0.06T} = ...$ (it cannot be for awrt 29.9). Do not allow $...T\mathrm{e}^{0.04T} = ...T\mathrm{e}^{-0.02T}$

A1ft: $0.8\mathrm{e}^{0.04T} = "4.8"\mathrm{e}^{-0.02T}$. Follow through on their positive value for *p*.

May be implied by further work which is not awrt 29.9.

dM1: Rearranges their $\frac{\mathrm{d}V_A}{\mathrm{d}t} = -\frac{\mathrm{d}V_B}{\mathrm{d}t}$ which cannot be the original functions to $r\mathrm{e}^{\pm 0.06T} = s$ where $r\times s > 0$

$\frac{\mathrm{d}V_B}{\mathrm{d}t} = "4.8"\mathrm{e}^{-0.02T} \Rightarrow 0.8\mathrm{e}^{0.04T} = "4.8"\mathrm{e}^{-0.02T}$ is allowed (see note at the start of (b).

Condone slips. It is dependent on the first method mark.

If they take lns of both sides first

e.g. $0.8\mathrm{e}^{0.04T} = "4.8"\mathrm{e}^{-0.02T} \Rightarrow \ln 0.8 + 0.04T = \ln"4.8" - 0.02T$ they need to rearrange to $\pm 0.06T = ...$ by adding / subtracting.

A1cso: awrt 29.9 provided evidence of solving $0.8e^{0.04T} = 4.8e^{-0.02T}$ is seen. i.e. they cannot proceed from $e^{\pm 0.06T} = ...$ to the awrt 29.9 in one step. We must see either an expression for t or e.g. taking lns of both sides.

Condone $(T =)\dfrac{50\ln 6}{3}$ or others in the form $(T =)\ a\ln b$ where a and b are rational constants e.g. $\dfrac{\ln 6}{0.06}$. It cannot be scored for an expression.

Question	Scheme	Marks	AOs
12(a)	(Surface area $=$) $2xy+\dfrac{\pi x^2}{4}$	B1	1.1b
	$2xy+\dfrac{\pi x^2}{4}=100\Rightarrow y=\dfrac{400-\pi x^2}{8x}=\dfrac{50}{x}-\dfrac{\pi x}{8}$	M1	3.4
	$(P=)\ 2x+4y+\dfrac{2\pi x}{4}$	B1	1.1b
	$(P=)\ 2x+4\left(\dfrac{400-\pi x^2}{8x}\right)+\dfrac{2\pi x}{4}$	M1	3.4
	$P=2x+\dfrac{200}{x}$ *	A1*	2.1
		(5)	
(b)	$\left(\dfrac{dP}{dx}=\right)\ 2-200x^{-2}$	M1 A1	1.1b 1.1b
	$2-200x^{-2}=0\Rightarrow x=...$	dM1	3.1b
	$x=10$	A1	1.1b
		(4)	
(c)	$\left(\dfrac{d^2P}{dx^2}=\right)"400"x^{-3}\Rightarrow"400"\times10^{-3}>0$	M1	1.1b
	e.g. $\dfrac{d^2P}{dx^2}(=0.4)>0$ hence minimum (perimeter)	A1	2.4
		(2)	
(d)	e.g. $y=\dfrac{400-\pi\times"10"^2}{8\times"10"}$	M1	3.4
	e.g. $y=1.07$ (m) so yes this would be suitable	A1	2.2a
		(2)	
		(13 marks)	

Notes

(a) **Note that different sections of the perimeter may be completed separately and brought together in a final line. Most marks will only be scored at this point – send to review if unsure.**

B1: Correct expression for the surface area in terms of x and y only (may be in an equivalent form). May be implied by their equation set equal to 100 or their rearranged form.

M1: Sets their expression in x and y equal to 100 and rearranges to make y (or $2y$ or $4y$) the subject. Do not be concerned by the mechanics of their rearrangement. May be implied by further work or can be scored for a different valid substitution into their expression for the perimeter.

B1: Correct expression for the perimeter in terms of x and y (may be in an equivalent form) and may be implied by an expression for the perimeter in terms of x if they have substituted in for their y straight away (which may be incorrect or have been rearranged incorrectly)

M1: Attempts to substitute their y into their perimeter to produce an expression or equation in just x. Condone invisible brackets for this mark. Condone slips provided the intention is clear.

A1*: $P = 2x + \dfrac{200}{x}$ cso (Condone the omission of $P =$ on the final line if it is seen on an earlier line of working) Allow Perimeter =
Do not withhold this mark if missing/invisible brackets are recovered in their working.

(b)

M1: Attempts to differentiate the given expression for P achieving an answer of the form $A \pm Bx^{-2}$
Condone candidates who do not achieve the given answer but their derivative differentiates to the required form to score this mark (and possibly dM1)

A1: $2 - 200x^{-2}$ o.e.

dM1: Sets their derivative of the form $A - Bx^{-2}$, $A \times B > 0$ equal to 0 and rearranges to find a value for x. It is dependent on the previous method mark. Condone slips in their rearrangement. May proceed directly to the answer.

A1: 10 cao (provided a correct derivative is seen) ± 10 is A0 isw if they attempt to find P

(c) Note that attempts only evaluating the gradient either side of $x = 10$ is M0A0

M1: Finds $\dfrac{d^2P}{dx^2}$ of the form Ax^{-3} o.e. and either considers the sign or evaluates for their positive x

A1: Requires

- $\left(\dfrac{d^2P}{dx^2} =\right) 400x^{-3}$ (condone this mark if $\dfrac{d^2y}{dx^2}$ is written or any other incorrect notation)
- reference to $400x^{-3}$ being > 0 for x > 0, or by using either a correct calculation (0.4 o.e.), a correct numerical expression or the algebraic expression and referencing that it is > 0 for $x = 10$
- correct conclusion e.g. hence min, shown, tick, QED
  Condone "minimum value of x"

(d)

M1: Either

- substitutes their value of x into any of their equations involving y or their expression for y from part (a) (unless restarted) to find a value for y
- uses their value of x to find their minimum value of P using the given expression for P, and then uses their x and their P to find a value for y
- attempts to find the value of x when $y = 1$ (allow to be solved directly from a calculator once a three-term quadratic has been formed)
- attempts to find the value of P using the given expression for P, their value for x and $y = 1$

Condone slips provided the intention is clear of their intended method.

A1: awrt 1.1 m and concludes would be suitable (yes is sufficient or a tick)
Note that "yes suitable because e.g. $1.07 > 0$ is A0" (it had to be greater than 1)

If they find the value for x when $y = 1$ it requires a correct comparison of awrt 10.1 with 10 so yes suitable

If they find the value of P using $x = 10$ and $y = 1$ it requires a correct comparison of awrt 39.7 with 40 so yes suitable

Question	Scheme	Marks	AOs
13(a)	$7\sin^2\theta - 4\sin\theta\cos\theta = 4$ $\Rightarrow 7\tan^2\theta - 4\tan\theta = \dfrac{4}{\cos^2\theta}$	M1	1.1b
	$\Rightarrow 3\tan^2\theta - 4\tan\theta + \dfrac{4\sin^2\theta}{\cos^2\theta} - \dfrac{4}{\cos^2\theta} = 0$	dM1	1.1b
	$\dfrac{4\sin^2\theta - 4}{\cos^2\theta} = -4\dfrac{\cos^2\theta}{\cos^2\theta} = -4 \Rightarrow 3\tan^2\theta - 4\tan\theta - 4 = 0$ *	ddM1 A1*	2.1 1.1b
		(4)	
(b)	Roots: $-\dfrac{2}{3}$, 2	B1	1.1b
	$\tan^{-1}\left("-\dfrac{2}{3}"\right) = ...$ or $\tan^{-1}("2") = ...$	M1	1.1b
	Two of awrt 63, awrt 146, awrt 243, awrt 326	A1	1.1b
	All four of awrt 63.4, awrt 146.3, awrt 243.4, awrt 326.3	A1	1.1b
		(4)	
(c)	awrt 735.9	B1	2.2a
		(1)	
			(9 marks)

Notes

(a) There are various ways of proceeding to the given answer. May work in θ or another variable which is acceptable. Condone a mixture of θ and e.g. x in the same equation or expression within their working.

Condone poor notation e.g. $\cos\theta^2$, $\sin^2 + \cos^2 = 1$ provided the intention is clear.

You may see multiple attempts so score the attempt which is most complete

M1: Score for any one of the following (somewhere in their same attempt which may already have errors in their working)

- divides **all** terms by $\cos^2\theta$
- uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ (or $\sec\theta = \dfrac{1}{\cos\theta}$)
- uses the identity $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$ (or $\pm 1 \pm \tan^2\theta = \pm\sec^2\theta$)

dM1: Score for any two of the following (somewhere in their same attempt which may already have errors in their working)

- divides **all** terms by $\cos^2\theta$
- uses $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ (or $\sec\theta = \dfrac{1}{\cos\theta}$)
- uses the identity $\pm\sin^2\theta \pm \cos^2\theta = \pm 1$ (or $\pm 1 \pm \tan^2\theta = \pm\sec^2\theta$)

It is dependent on the previous method mark.

ddM1: A full attempt to rearrange the equation to the form $a\tan^2\theta + b\tan\theta + c = 0$. It is dependent on the previous two method marks. When rearranging condone sign slips or a missing trailing bracket only.

A1*: Proceeds to the given answer with all steps shown and no errors other than what is condoned (see above). Condone a missing trailing bracket only.
The final answer must be in terms of θ and written using the correct notation.

Note they may work in reverse from $3\tan^2\theta - 4\tan\theta - 4 = 0$ to $7\sin^2\theta - 4\sin\theta\cos\theta = 4$ or meet somewhere in the middle. In these cases there must be some minimal conclusion e.g. QED or tick
The first two method marks should be applied in the same way as the main scheme.
The dM1 mark will be scored for reaching a form of $\sin\theta(d\sin\theta + e\cos\theta) = f$

(b)	**Answers with no working scores 0 marks.** **Beware of correct angles following incorrect roots which cannot score the A marks**
B1:	Roots: $-\frac{2}{3},\ \ 2$
M1:	Attempts to find at least one of the angles for one of their roots. This may be implied by a correct answer. You may need to check this on your calculator.
A1:	Two of awrt 63, awrt 146, awrt 243, awrt 326 (**must come from a correct root**)
A1:	All four of awrt 63.4, awrt 146.3, awrt 243.4, awrt 326.3 and no others in the range Withhold the final mark if they solve for 4α (and subsequently divide their angles by 4)
(c)	
B1:	awrt 735.9

Question	Scheme	Marks	AOs
14	Sets up the proof by exploring when $n = 2k$ or $n = 2k+1$ e.g. $(2k)^2 + 5(2k) = ...k^2 + ...k$ **or** $(2k+1)^2 + 5(2k+1) = ...k^2 + ...k + ...$	M1	1.1b
	e.g. $4k^2 + 10k$ **or** $4k^2 + 14k + 6$ and shows or gives a reason why the expression is even (see notes)	A1	2.2a
	Explores when $n = 2k$ **and** $n = 2k+1$ eg $(2k)^2 + 5(2k) = ...k^2 + ...k$ **and** $(2k+1)^2 + 5(2k+1) = ...k^2 + ...k + ...$	dM1	2.1
	e.g. $4k^2 + 10k$ **and** $4k^2 + 14k + 6$ and shows or gives a reason why both of the expressions are even (see notes) hence $n^2 + 5n$ is even for all $n (\in \mathbb{N})$ (or equivalent)	A1*	2.4
			(4 marks)

Notes

Main scheme algebraic method using e.g. $n = 2k$ and $n = 2k \pm 1$
You will need to look at both cases and mark the one which is fully correct first.
Allow a different variable to *k* and may be different letters for odd and even.
Condone use of *n* as a variable for the first three marks.
There should be no errors in the algebra for the A marks but allow e.g. invisible brackets to be "recovered".

M1: Sets up the proof by exploring when *n* is odd or even e.g. $n = 2k$ or $n = 2k+1$ (or equivalent), and either expands and achieves a quadratic expression (which may be unsimplified) or allow to factorise e.g. $2k(2k+5)$ or e.g. $(2k+2)(2k+7)$

Condone slips. e.g. $2k(2k+5) = 2k^2 + 10k$ or slips when collecting terms.

A1: Correct quadratic expression (which may be unsimplified) for $n^2 + 5n$ for either odds or evens and shows or gives a reason why the expression is even. They must have fully multiplied out or the quadratic expression must be factorised completely.

e.g. $4k^2 + 10k = 2(2k^2 + 5k)$ (which is even)

e.g. $4k^2 + 14k + 6 = 2(2k^2 + 7k + 3)$ (which is even)

e.g. $\dfrac{4k^2 + 10k}{2} = 2k^2 + 5k$ (hence even)

e.g. "2 is a factor of both terms", "all divisible by 2" (so even)
If a reason is given as well as an algebraic expression it must be correct
e.g. $4k^2 + 10k = 2(2k^2 + 5k)$ so even as can be multiplied by 2 can score M1A1
but $\dfrac{4k^2 + 10k}{2} = 2k^2 + 5k$ so it can be divided by 2 so even is M1A0 (needs to say divisible by 2)
Do not isw if they simplify their quadratic incorrectly.
Note that they do not have to state that the expression is even if they conclude for all cases at the end.

dM1: Explores when *n* is odd and when *n* is even leading to two quadratic expressions (may be factorised) for when $n = 2k$ and $n = 2k+1$ (or equivalent) (see first M1 for guidance)

A1*: Requires

- correct quadratic expression for n^2+5n for both odds and evens
- shows or gives a reason for each why the expressions are even (see first A1 for guidance)
- makes a concluding overall statement. "Hence n^2+5n is even for all $n(\in \mathbb{N})$" (or equivalent).

Note that if they have stated for each separate case that the expression is even then allow minimal statements of "hence proven", "statement proved", "QED", tick

Do not isw this mark if they simplify their quadratic incorrectly.

	n^2+5n
$2k-3$	$4k^2-2k-6$
$2k-2$	$4k^2+2k-6$
$2k-1$	$4k^2+6k-4$
$2k$	$4k^2+10k$
$2k+1$	$4k^2+14k+6$
$2k+2$	$4k^2+18k+14$
$2k+3$	$4k^2+22k+24$

Alternative methods:

Algebraic with logic example

e.g. $n^2+5n=n(n+5)$

When n is odd then $n+5$ is even so odd x even is even

When n is even then $n+5$ is odd so even x odd is even

Both cases must be considered to score any marks and scores SC 1010 if fully correct

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Further Maths method (proof by induction) – you may see these but please send to review for TLs or above to mark

M1: Assumes true for $n=k$, substitutes $n=k+1$ into n^2+5n, multiplies out the brackets and attempts to simplify to a quadratic expression (which may be unsimplified)

e.g. k^2+7k+6 Condone arithmetical slips

A1: $\left(f(k+1)=3k^2+3k+1+6(k+1)=\right)\ k^2+5k+2k+6=f(k)+2(k+3)$

which is even + even = even

dM1: Attempts to substitute $n=1 \Rightarrow 1^2+5\times1=6$ (which is true) (Condone arithmetical slips evaluating)

A1*: Explains that

- it is true when $n=1$
- if it is true for $n=k$ then it is true for $n=k+1$
- therefore it is true for all $n(\in \mathbb{N})$

Solutions via just logic (no algebraic manipulation) scores 0 marks.

e.g.

If n is odd, then n^2+5n is $odd^2+odd\times odd=odd+odd=even$

If n is even, then n^2+5n is $even^2+odd\times even=even+even=even$

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